

ISC.

(1) SUBMULTIPLE
ANGLES

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PHONE: 2411-2225.Problem Sheet - 321. Using the value of $\cos 45^\circ$ find the values of:-

$$\sin 22\frac{1}{2}^\circ, \cos 22\frac{1}{2}^\circ, \text{ and } \tan 22\frac{1}{2}^\circ.$$

2. From $\sin 30^\circ$ find the value of $\sin 15^\circ$.3. Prove that $4 \sin 27^\circ = \sqrt{5+\sqrt{5}} - \sqrt{3-\sqrt{5}}$.4. If $\sin \theta = -\frac{4}{5}$ and $180^\circ < \theta < 270^\circ$, find the values of:-

$$\sin \frac{\theta}{2} \text{ and } \cos \frac{\theta}{2}.$$

5. If $360^\circ < \theta < 450^\circ$ and $\cos \theta = \frac{7}{25}$ find the values of
 $\sin \frac{\theta}{2}$ and $\cos \frac{\theta}{2}$.6. If $\sin \alpha = \frac{4}{5}$, $\cos \beta = \frac{12}{13}$, and α, β are two acute angles,
find the values of $\sin \frac{\alpha-\beta}{2}$, and $\cos \frac{\alpha-\beta}{2}$ 7. If $\theta = -240^\circ$, is the following relation true? Justify,

$$2 \sin \frac{\theta}{2} = -\sqrt{1+\sin \theta} + \sqrt{1-\sin \theta}$$

8. Show that $\sec \alpha + \tan \alpha = \tan \left(\frac{\pi}{4} + \frac{\alpha}{2} \right)$ 9. Show that $\frac{\sec \theta - \tan \theta}{\sec \theta + \tan \theta} = \cot^2 \left(\frac{\pi}{4} + \frac{\theta}{2} \right)$ 10. Without using table prove that, $\sin 12^\circ \sin 48^\circ \sin 54^\circ = \frac{1}{8}$.11. Prove that $\cos \frac{\pi}{32} = \frac{1}{2} \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2}}}}}$. Hence find $\sin \frac{\pi}{32}$

(2)

- (12) Prove that $\tan 7\frac{1}{2}^\circ = \sqrt{6} - \sqrt{3} + \sqrt{2} - 2$.
- (13) Show that $\tan 6^\circ \tan 42^\circ \tan 66^\circ \tan 78^\circ = 1$
- (14) Find the value of $\cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \frac{5\pi}{8} + \cos^4 \frac{7\pi}{8}$
- (15) Show that $(\sin^2 24^\circ - \sin^2 6^\circ)(\sin^2 42^\circ - \sin^2 12^\circ) = \frac{1}{16}$.
- (16) If $\tan \frac{\theta}{2} = \sqrt{\frac{1-e}{1+e}}$ prove that,

$$\cos \phi = \frac{\cos \theta - e}{1 - e \cos \theta}$$
- (17) If $\cos \alpha = \cos \beta \cos \phi = \cos \beta' \cos \phi'$ and $\sin \alpha = 2 \sin \frac{\phi}{2} \sin \frac{\phi'}{2}$
 show that $\tan^2 \frac{\alpha}{2} = \tan^2 \frac{\beta}{2} \tan^2 \frac{\beta'}{2}$
- (18) If $\frac{1 - \sin \alpha}{1 + \sin \alpha} = \tan^2 \frac{\beta}{2}$ show that $\alpha + 2\beta = 180^\circ$.
- (19) If $\theta = 400^\circ$ determine the correct signs in the following identity :- $\sin \frac{\theta}{2} = \pm \frac{1}{2} \sqrt{1 + \sin \theta} \pm \frac{1}{2} \sqrt{1 - \sin \theta}$.
- (20) If $\tan \theta = \frac{\sin \alpha \sin \beta}{\cos \alpha + \cos \beta}$, show that, one value of $\tan \frac{\theta}{2}$
 is $\tan \frac{\alpha}{2} \tan \frac{\beta}{2}$